Revelation of Optimum Modes of Ultrasonic Influence for Atomization of Viscous Liquids by Mathematical Modelling

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Abstract **– In the article the process of cavitation low-frequency (up to 250 kHz) ultrasonic atomization of viscous liquids in a layer is investigated. It takes place with entering of acoustic energy to working zone through liquid. To reveal optimum modes of ultrasonic influence depending on physical properties of atomized liquid (viscosity, surface tension, etc.) the model describing stepwise transformation of mechanical vibration energy of ulrtasonic frequency into energy of capillary waves providing the formation of drops was proposed and developed. For the first time we offer theoretical explanation of essential dependence of drop diameter on vibration amplitude of spraying surface based on changes of mean thickness of ridges of capillary waves according to their amplitude due to occurence of nonlinear effects. Obtained results can be a base for the design of specialized ultrasonic atomizers of liquids with high viscosity for the formation of aerosols with specified productivity and dispersed features.**

Index Terms **– Ultrasound, atomization, aerosol.**

I. INTRODUCTION

THE ATOMIZERS of different liquids are the base of a great deal of technological processes at the productions concerning to high-tech sector of the economy [1].

They are the atomizers intending for the operation as a part of spectral analysis installation (for the supply of sol to the source) in the aircraft propulsion engineering, the systems of photoresist sputtering in the microelectronic industry, polishing liquids in the optic-electronic instrument engineering, the devices of coating deposition at the production of medical equipment and implants, liquid atomizers at the production of the functional nanomaterials, etc.

Among great varieties of different atomizers the most effective are ultrasonic ones [1], as they have unique advantages: low power-consuming and high productivity of the process; the possibility of fine-disperse and monodisperse atomization; the absence of necessity of application of spraying agent; the possibility of atomization of high-viscous liquids without adding of solvents, atomization of metal melts, formation of high-quality coatings; the presence of circulating currents in the liquid drops contributing acceleration of heat exchange and mass transfer processes at the surface of the drop.

The diversity of applications of ultrasonic atomizers demands analysis of their operation modes according to the set of optimality criteria, each of them is individual for the certain process.

For instance, at the preliminary atomization of liquid-drop aerosol it is necessary to provide maximum dispersion of distribution function of particle size for the coagulation of solid particles, as polydisperse aerosol is coagulated more effectively than monodisperse one according to orthokinetic mechanism of particles interaction. For the deposition of uniform coatings it is necessary to provide minimum possible diameter of formed drops with its minimum possible roof-mean-square deviation from mean value with required productivity.

Thus, solving different technological problems there is a need to provide ultrasonic atomization of liquid with maximum effectiveness of the process. The effectivenss of the process is detemined by the following characteristics:

– disperse composition of the aerosol spray (mean diameter of the drop and aerosol dispersivity);

– productivity of aerosol formation with specified spray.

As physical phenomena originating and taking place under the influence of ultrasonic vibrations in a thin layer at the ultrasonic atomization are transient and they occur at the microlevel, their experimental investigation for revealation of optimum modes of the influence is complicated. However existing scientific approaches based on the application of mathematical models let describe these phenomena. That is why, at the first stage of the studies the investigations by mathematical model approach of physical phenomena originating under the influence of ultrasonic vibrations taking into account their interference and infuencing factors, which were not studied before, for the revealation of optimum parameters of ultrasonic influence upon thin layer of liquid are the most worthwhile.

II. PROBLEM DEFINITION

At the ultrasonic influence on thin layers of liquids the vibrations of radiating surface of the ultrasonic transducer generate variable acoustic pressure in a liquid layer, which at the achievement of specified values can lead to the formation of cavitation pockets (bubbles) in liquid, which possess a store of energy accompanying with its instantaneous release at the collapse of bubble.

The occurrence of the cavitation causes the appearance of shocj wave, which at the fulfillment of specified conditions (thickness of the layer, viscosity and surface tension of liquid, amplitude of ultrasonic influence) leads to the formation of surface effects at the interface of liquid and gaseous medium – capillary waves. It takes place according to the widespread cavitation-wave theory, which was first proposed by Boguslavsky and Eknadiosyants [2] and further developed by Novitsky [3].

Capillary waves provide considerable increase of free interface of liquid with gaseous medium, that can have important applied significance, e.g. for the intensification of absorption and heat transfer processes. When capillary waves are achived threshold amplitude, they form separate drops that provides ultrasonic atomization of liquids.

The attempt of complete study of ultrasonic atomization process was for the first time proposed in the paper [4], it gave stage-by-stage description of drop formation under the ultrasonic influence. However it does not allowed to determine the disperse characteristics of formed aerosol under influencing parameters, such as frequency and vibration amplitude, surface tension and density of atomizing liquid. The analysis of a great number of papers of modern Russian and foreign researchers shows, that at present there is no theoretical explanation of the dependence of the diameter of formed drops on the vibration amplitude at low-frequency atomization, and all known models of formation of capillary waves on the liquid surface do not take into consideration its viscosity. Moreover at present time there is no theoretical base for the determination of the characteristics of the spray.

To reveal optimum modes of ultrasonic influence on thin liquid layer providing its atomization with the formation of drops with specified disperse characteristics and productivity the model was developed. It descibes the process of drop formation consisting of following stages:

1. Determining of the pressure in the shock wave front on the free surface at the cavitation bubble collapse on vibrating surface.
2. Determining of the profile of formed capillary waves.
3. Determining of the mean diameter of formed drops.
4. Determining of the productivity of the atomization.
5. Determining of the speed of droplet detachment and the height of the spray.

To determine all these dependences further stage-by-stage investigation of the process is carried out by the mathematical modelling.

III. The Analysis of Cavitation Development In the Layer of Atomized Liquid

The determining of acoustic pressure in the layer of atomizde liquid depending on the thickness of this layer is carried out on the base of the analysis of the system of Navier – Stokes linearized equations in one-dimensional case under the assumption of the absence of interference maximum of the short-distance field of the surface of the ultrasonic transducer.

As acoustic pressure achieves maximum value on the surface of the transducer, the development of the cavitation in the layer bordering with the surface of the ultrasonic transducer [1] is of great interest. The amplitude of acoustic pressure directly at atomizing surface in the case of thin layers is kh << 1:

$$|P(0)| = \rho_0 \omega^2 A h$$

where $|P(0)|$ is an amplitude of the acoustic pressure near the surface of working tool, ρ_0 is an equilibrium liquid density, ω is a circular frequency of acoustic vibrations, A is a vibration amplitude of of the surface of working tool, h is a thickness of liquid layer.

The most important parameter of the cavitation is a size of cavitation bubble in the stage of its maximum expansion. All other significant parameters of cavitation – the speed of collapse and the pressure in the shock wave front formed during the cavitation bubble collapse are connected with the first parameter.

As modelling of dynamics of several cavitation bubbles is attended by mathematical dissiculties, it is worthwhile to consider the model of expansion and collapse of single cavitation bubble.

The dynamics of cavitation buble is expressed by the Kirkwood-Bethe equation [5], i.e. differential equation relatively to the radius of cavitation bubble as a function from time at known acoustic pressure (1):

$$R\left(1-\frac{\frac{\partial R}{\partial t}}{C}\right)\frac{\partial^2 R}{\partial t^2}+\frac{3}{2}\left(1-\frac{\frac{\partial R}{\partial t}}{3C}\right)\left(\frac{\partial R}{\partial t}\right)^2=\left(1+\frac{\frac{\partial R}{\partial t}}{C}\right)H+\frac{R}{C}\left(1-\frac{\frac{\partial R}{\partial t}}{C}\right)\frac{\partial H}{\partial t}, \quad (1)$$

where C is a local velocity of sound in liquid, H is an enthalpy of liquid, R is a radius of the bubble.

To determine the value of pressure amplitude in the shock wave front formed at the cavitation bubble collapse and spreading in liquid it is possible to apply the results of studies of underwater explosion phenomemenon carried out by Cole [3] on the model, which represents the explosion as a as an extension of sphere under the influence of gas filling it. According to the theory of underwater explosion the pressure of shock wave at the collapse of cavitation bubble in thin layer of liquid at the interface of “liquid-air” is several tens of atmospheres.

The pressure of shock wave lets determine the profile of formed capillary waves.

IV. Determining of The Profile of Formed Capillary Waves

The dynamics of liquid at the formation of capillary waves is described by well-known equations of Navier-Stokes connecting velocity fields and pressure of liquid medium:

$$div\,\mathbf{u} = 0 \quad (2)$$

$$\rho\left(\frac{\partial \mathbf{u}}{\partial t}+(\mathbf{u},\nabla)\mathbf{u}\right) = -\nabla p + \mu\Delta\mathbf{u} \quad (3)$$

Boundary conditions on the free surface are the following:

a) dynamic condition for tensions on the free surface

$$-p\mathbf{n}+2\mu\sum_{i=1}^{2}\sum_{j=1}^{2}\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)n_i\mathbf{e}_j=\sigma\frac{\frac{\partial^2\xi}{\partial x^2}}{\left(1+\left(\frac{\partial\xi}{\partial x}\right)^2\right)^{\frac{3}{2}}}\mathbf{n} \quad (4)$$

$$u_1 = v;\, u_2 = u;\, x_1 = x;\, x_2 = y$$

b) kinematic condition for free surface displacement

$$\frac{\partial \xi}{\partial t}+\frac{\partial \xi}{\partial x}v=u \qquad (5)$$

c) conditions at infinity (at sufficiently large distance from free surface of liquid)

$$p\big|_{y=-\infty}=0 \qquad (6)$$

$$\mathbf{u}\big|_{y=-\infty}=0 \qquad (7)$$

As the amplitude of capillary wave is enough for atomization, comparable with the wave length and is no less than 0.73 of its length [5], the deviation of the profile of capillary wave from sinusoidal form due to nonlinear effect will take place.

In view of nonlinear development of capillary waves surface wave at high amplitudes will consist of the sum of several harmonics, each of which is appropriate infinitesimal order $\xi^{(n)}$ is *n*-th harmonic of free surface displacement, $p^{(n)}$ – *n*-th harmonic of liquid pressure, $\mathbf{u}^{(n)}$– *n*-th harmonic of liquid velocity, each harmonic will have different length.

Boundary conditions for the first harmonic are following:

a) kinematic condition

$$\frac{\partial \xi^{(1)}}{\partial t}=u^{(1)}$$

b) dynamic conditions

$$-p^{(1)}+4\mu\frac{\partial u^{(1)}}{\partial y}=\sigma\frac{\partial^2 \xi^{(1)}}{\partial x^2} \qquad (9)$$

$$\frac{\partial v^{(1)}}{\partial y}+\frac{\partial u^{(1)}}{\partial x}=0 \qquad (10)$$

Linearized system of equations of Navier-Stokes connecting velocity fields and medium pressure of the first order is represented in a following way:

$$div\,\mathbf{u}^{(1)}=0 \qquad (11)$$

$$\rho\frac{\partial \mathbf{u}^{(1)}}{\partial t}=-\nabla p^{(1)}+\mu\Delta\mathbf{u}^{(1)} \qquad (12)$$

The solution of the equation for velocity is:

$$\mathbf{u}^{(1)}=\nabla\varphi^{(1)}+\mathbf{w}^{(1)} \qquad (13)$$

where $\varphi^{(1)}$ is a velocity potential of liquid motion, $\mathbf{w}^{(1)}$ is a solenoidal component of velocity of liquid motion, $\mathbf{u}^{(1)}$is a velocity of liquid motion.

The velocity potential is defined by the following relation:

$$\varphi^{(1)}=-\frac{1}{\rho}\int p^{(1)}\partial t \qquad (14)$$

Taking into account continuity equation the velocity potential satisfies Laplace's equation:

$$\Delta\varphi^{(1)}=0 \qquad (15)$$

from the relations (14-15) and the equations (11-12) it results, that solenoidal vector satisfies following system of equations:

$$\mathbf{div}\,\mathbf{w}^{(1)}=0 \qquad (16)$$

$$\rho\frac{\partial \mathbf{w}^{(1)}}{\partial t}=\mu\Delta\mathbf{w}^{(1)} \qquad (17)$$

Under the assumption that capillary wave has length λ we use following notation (18):

$$k=\frac{2\pi}{\lambda} \qquad (18)$$

where *k* is a wave number of the capillary wave.

The solution of Laplace's equation for the velocity potential can be found in the following way (19):

$$\varphi^{(1)}=Be^{ky}e^{\pm ikx-i\omega t} \qquad (19)$$

where B is a constant defined by boundary conditions.

The solution of the system of equations (16-17) for solenoidal vector can be found as follows:

$$\mathbf{w}^{(1)}=\mathbf{W}_A^{(1)}e^{k\sqrt{1-i\frac{\rho\omega}{\mu k^2}}y}e^{\pm ikx-i\omega t}=\begin{pmatrix}W_{A1}^{(1)}\\ W_{A2}^{(1)}\end{pmatrix}e^{k\sqrt{1-i\frac{\rho\omega}{\mu k^2}}y}e^{\pm ikx-i\omega t} \qquad (20)$$

where $W_{A1}^{(1)}$, $W_{A2}^{(1)}$ are constants, which are connected with each other through the constant coefficient.

The free surface displacement is represented as follows:

$$\xi^{(1)}=H^{(1)}e^{\pm ikx-i\omega t} \qquad (21)$$

where $H^{(1)}$ is an amplitude of the first harmonic of the capillary wave.

Disturbance of liquid pressure is expressed by the velocity potential as follows (22):

$$p^{(1)}=-\rho\frac{\partial\varphi^{(1)}}{\partial t}=i\omega\rho Be^{ky}e^{\pm ikx-i\omega t} \qquad (22)$$

After substitution of the expressions for solenoidal and potential vector component of velocity (19-20), mixing of free surface (21) and medium pressure (22) in the kinematic and dynamic boundary conditions (8-10), and expressing amplitude mixing of free surface displacement $H^{(1)}$ through the constants B and C as a result we have a system of equations joining together constants B, C and wave number of capillary wave (23, 24):

$$\omega^2\rho B+4i\omega\mu\left(k^2B+k\sqrt{1-i\frac{\rho\omega}{\mu k^2}}C\right)=\sigma k^2(kB+C) \qquad (23)$$

$$2ik^2B+ik\left(1-i\frac{\rho\omega}{\mu k^2}\right)C+ikC=0 \qquad (24)$$

From the system of equations (23-24) we get complex algebraic equation, which allows to determine explicitly wave number of capillary wave:

$$\omega^2\rho\left(1-i\frac{\rho\omega}{2\mu k^2}\right)\frac{1}{k}-4i\omega k\mu\left(-1+i\frac{\rho\omega}{2\mu k^2}+\sqrt{1-i\frac{\rho\omega}{\mu k^2}}\right)=-i\sigma\frac{\rho\omega}{2\mu} \qquad (25)$$

Obtained wave number *k* allows to determine the length of the first harmonic of the capillary wave.

The dependences of capillary wave length on liquid viscosity at different frequencies of acoustic vibrations are shown in Fig. 1.

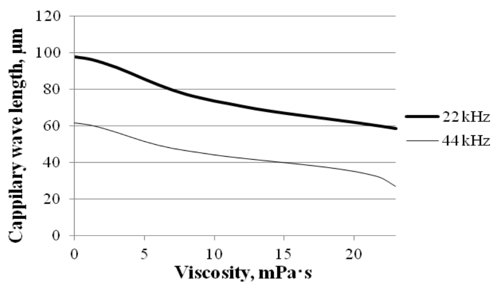


Fig. 1. The dependences of capillary wave length on liquid viscosity at different frequencies of acoustic vibrations.

Thus developed model lets define capillary wave length taking into consideration viscosity as at low also at high Reynolds numbers.

To determine the amplitude of capillary wave the approach based on the application of energy conservation law is used. Using the energy conservation law it is proposed, that all energy of shock wave is transferred to the energy of capillary wave on the surface of liquid.

Sum kinetic energy of liquid at the formation of capillary wave is defined by the following expression:

$$E_{\text{cap}} = \frac{\rho}{2}\int_{0}^{\frac{\lambda}{2}}\int_{-\infty}^{0} {v_0}^2 r\cos^2(kr)e^{2kz}\partial z\partial r = \frac{\rho f^2\lambda^3 A^2}{16\pi}\left(\frac{\pi^2}{2}-\frac{1}{4}\right) \tag{26}$$

where ρ is a density of liquid, ω is a circular frequency, A is a vibration amplitude of capillary wave, λ is a length of capillary wave, v_0 is a vibrational speed of liquid.

At the next stage it is necessary to determine the enrgy of shock wave. The energy of shock wave is defined by the velocity of liquid motion gained at the time of passing of its front.

Velocity of liquid motion gained at the time during the action of shock wave can be formulated based on the expression (27):

$$v = \frac{P_m h}{\rho r^2}\theta \tag{27}$$

where P_m is a pressure of shock wave near the free surface, h is a thickness of liquid layer, r is a distance, at which velocity of liquid motion is calculated, Θ is a lifetime of impulse of the shock wave.

At the calculation of velocity of motion of free surface at the distance, which equals to the thickness of liquid layer the expression is transformed in a following way:

$$v = \frac{P_m}{\rho h}\theta \tag{28}$$

The energy of shock wave is completely converted into the energy of capillary wave and is defined according to this expression meaning summarized kinetic energy of liquid in the neighborhood of the front of shock wave (h, $h+c\Theta$):

$$E_{\text{sh}} = 2\pi h^2\int_{h}^{h+c\theta}\rho v^2\partial r = 2\pi c\frac{{P_m}^2}{\rho}\theta^3 \tag{29}$$

where P_m is a pressure in the front of shock wave, Θ is a lifetime of impulse of the pressure of shock wave, c is a velocity of sound in liquid.

The energy of shock wave is completely converted into the energy of capillary wave:

$$E_{\text{cap}} = E_{\text{sh}}$$

Hence following expression for the amplitude of capillary wave is obtained:

$$A = \frac{4\pi P_m\theta}{\rho f\lambda}\sqrt{\frac{c\theta}{\lambda\left(\frac{\pi^2}{4}-\frac{1}{8}\right)}} \tag{30}$$

Carried out preliminary calculations show, that the value of amplitude of capillary wave (at the vibration amplitudes of atomizing surface used in practice for atomization of liquids [1]) exceeds the value of 100 μm, this is comparable with its length.

At such significant amplitudes the profile of capillary wave deviates from sinusoidal form. As for further analysis of drop formation process it is necessary to obtain information on real profile of capillary wave, nonlinear perturbations of second-order of free surface displacement, i.e. we define the second harmonics of velocity field, liquid pressure and free surface displacement, are considered further.

To define the value of the second harmonic of capillary wave known solutions of Navier-Stokes equations with dynamic boundary conditions for the first harmonic of capillary wave are used. Required system of equations of Navier-Stokes for perturbation of second-order looks as follows:

$$div\,\mathbf{u}^{(2)} = 0 \tag{31}$$

$$\rho\left(\frac{\partial\mathbf{u}^{(2)}}{\partial t} + \left(\mathbf{u}^{(1)},\nabla\right)\mathbf{u}^{(1)}\right) = -\nabla p^{(2)} + \mu\Delta\mathbf{u}^{(2)} \tag{32}$$

Boundary conditions for perturbation of second-order are following:

а) dynamic conditions for tensions on free surface:

$$p^{(1)}\frac{\partial\xi^{(1)}}{\partial x} + 2\mu\left(\frac{\partial v^{(2)}}{\partial y} + \frac{\partial u^{(2)}}{\partial x} - 2\frac{\partial v^{(1)}}{\partial x}\frac{\partial\xi^{(1)}}{\partial x}\right) = -\sigma\frac{\partial^2\xi^{(1)}}{\partial x^2}\frac{\partial\xi^{(1)}}{\partial x} \tag{33}$$

$$-p^{(2)} + 2\mu\left(2\frac{\partial u^{(2)}}{\partial y} - \left(\frac{\partial v^{(1)}}{\partial y} + \frac{\partial u^{(1)}}{\partial x}\right)\frac{\partial\xi^{(1)}}{\partial x}\right) = \sigma\frac{\partial^2\xi^{(2)}}{\partial x^2} \tag{34}$$

б) kinematic condition for free surface displacement:

$$\frac{\partial\xi^{(2)}}{\partial t} + \frac{\partial\xi^{(1)}}{\partial x}v^{(1)} = u^{(2)} \tag{35}$$

в) conditions at infinity (at rather great distance from free surface of liquid)

$$\left.p^{(2)}\right|_{y=-\infty} = 0 \tag{36}$$

$$\left.\mathbf{u}^{(2)}\right|_{y=-\infty} = 0 \tag{37}$$

In this case the solution of system of equations of Navier-Stokes is following:

$$\mathbf{u}^{(2)} = \nabla\varphi_2 \tag{38}$$

After substitution of the expressions for the first harmonics of velocity potential and free surface displacement in boundary conditions following equations are obtained (39-41):

$$\frac{\partial\xi^{(2)}}{\partial t}(x,t) + k^2AB\sin^2(kx)e^{-2i\omega t} = -k^2AB\cos^2(kx)e^{-2i\omega t} + \frac{\partial\varphi_2}{\partial y}(x,0,t) \tag{39}$$

$$\frac{\partial\xi^{(2)}}{\partial t}(r,t) + k^2ABe^{-2i\omega t} = \frac{\partial\varphi_2}{\partial y}(x,0,t) \tag{40}$$

$$\frac{-i\omega kAB}{2}\left(1+\cos(kr)\right)e^{-2i\omega t} + \frac{\partial\varphi_2}{\partial t}(r,0,t) = \frac{\sigma}{\rho}\frac{\partial^2\xi^{(2)}}{\partial x^2} \tag{41}$$

where A is an amplitude of the first harmonic of free surface displacement, B is an amplitude of the first harmonic of velocity potential.

Taking into account the expression (38) for the function ξ the profile of capillary wave is represented by the following relation:

$$\xi(x,t) \approx A\sin(\omega t)\left(\cos(kx) - \left(\frac{Ak}{3}\cos(2kx) + 2Ak\right)\cos(\omega t)\right) \tag{42}$$

The next stage of the model investigation is determining of the diameter of aerosol drops on the base of known profile of capillary wave.

IV. Determining of The Diameter of Formed Drops

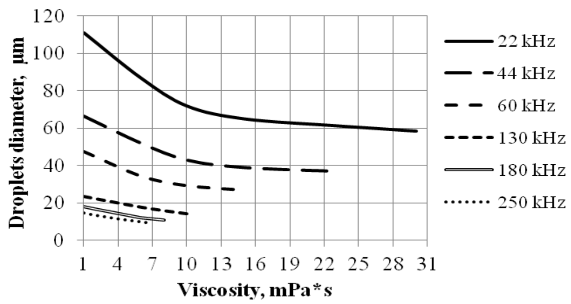
Obtained profile of capillary wave with regard to the second harmonic allows to define the diameter of formed aerosol drops. For this purpose on the base of obtained profile of capillary wave mean diameter is defined by the height $D_{cap.avg.}$ of local liquid rise:

$$D_{cap.avg.} = 2\frac{\int_{0}^{\frac{\lambda}{2}}\left(\xi(x)-\xi\left(\frac{\lambda}{2}\right)\right)\partial x}{\xi(0)-\xi\left(\frac{\lambda}{2}\right)} \quad (43)$$

If viscosity of atomized liquid is up to 30 cP and the surface tension is more than 0.03 N/m and the radius of the spray is more than 10 μm Reynolds number $\mathrm{Re} > 10$, the diameter of of drops depends only on mean thickness of capillary wave and it is defined by the following expression [7]:

$$D = 1{,}89 D_{cap.avg.} \quad (44)$$

Obtained dependences of drop diameter on the vibration amplitude for different in viscosity liquids and at different frequencies of acoustic influence are shown in Fig. 2, 3. All dependences are obtained at optimum thickness of liquid layer, i.e. the amplitude of capillary waves is maximum.



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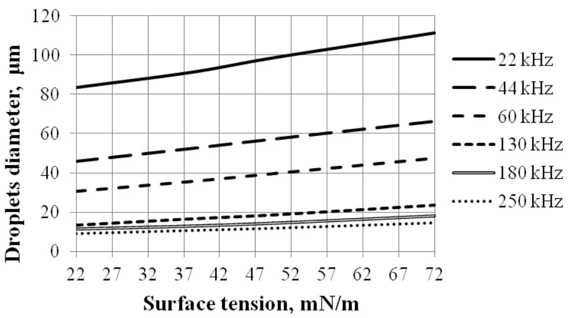

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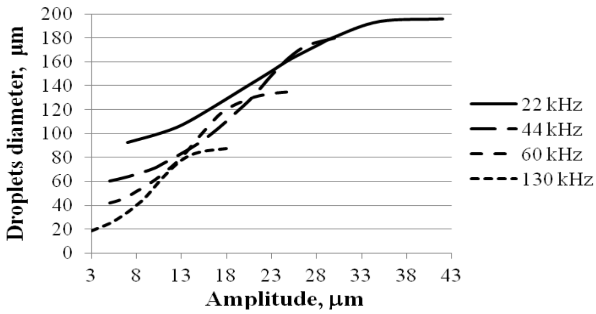
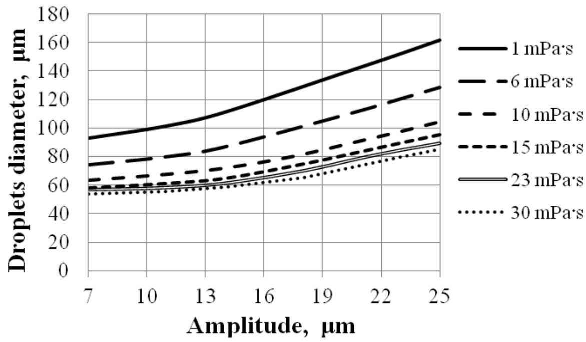


c)

Fig. 2. The dependences of drop diameter on different influencing factors: on viscosity (a), surface tension (b) and amplitude (c) at different frequencies.



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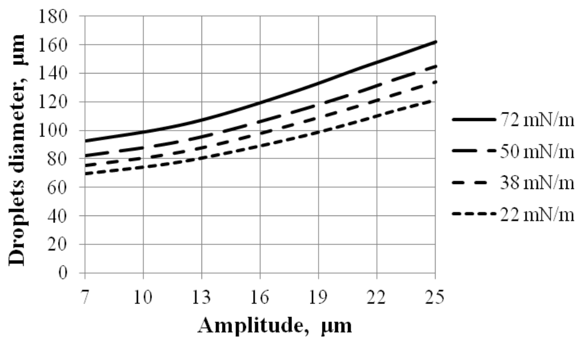


b)

Fig. 3. The dependences of drop diameter on amplitude at different viscosities (a) and surface tensions (b)

From obtained dependences it follows, that vibration amplitude of atomizing surface essentially influences on the diameter of formed liquid drops. Moreover the less is viscosity of atomized liquid, the stronger is the dependence of the diameter of formed drops on amplitude (in 1.4 times much as at the viscosity equals to $1 \cdot 10^{-3}$ Pa·s).

The presence of liquid viscosity leads to the reduction of the diameter of formed drops. The strongest influence is shown up to the value of viscosity of $10 \ldots 15 \cdot 10^{-3}$ Pa·s, after that the reduction of drop diameter practically stops.

It is determined, that the diameter of formed drops depends linearly directly proportional on surface tension of liquid, at that proportionality coefficient decreases with the rise of frequency.

It is revealed, that the dependence of the diameter of formed drops on vibration amplitude of atomizing surface has nonlinear character. At that speed of diameter rise of formed drops on the amplitude of ultrasonic influence is higher, when the frequency of ultrasonic vibrations is more. Obtained dependence proves the perspectiveness of application of studied method of ultrasonic liquid atomization in the range of high frequencies and the necessity of the development of new constructions of high-frequency ultrasonic vibrating systems for liquid atomization. It allows to form the aerosol with mean diameter of 10 μm and less.

In whole obtained dependences can be applied for the determining of technical requirements (frequency and amplitude of ultrasonic influence) to the atomizers providing the formation of aerosol with specified characteristics.

IV. Determining of Atomization Productivity

At the next stage of model study it is necessary to determine the productivity of atomization. As it was mentioned at the begining the formation of drops occured from capillary wave crest, which formed above collapsing cavitation bubble. At that in time, which equals to the period of ultrasonic vibrations, one capillary wave is formed under the cavitation bubble. From one capillary wave several drops may be formed, which summarized volume does not exceed the volume of liquid included inside the crest of capillary wave. Precise amount of liquid turned into drops from one capillary wave is calculated by the introduction of the coefficient *a*:

$$V = a\frac{\lambda^2 A}{2\pi}\left(\frac{\pi^2}{2} - 2\right) \qquad (45)$$

Where A is an amplitude of capillary wave; λ is a length of capillary wave; *a* is a correction coefficient taking into account volume fraction of capillary wave splitting into drops.

The amplitude and length of capillary wave were defined at the previous stage of model study.

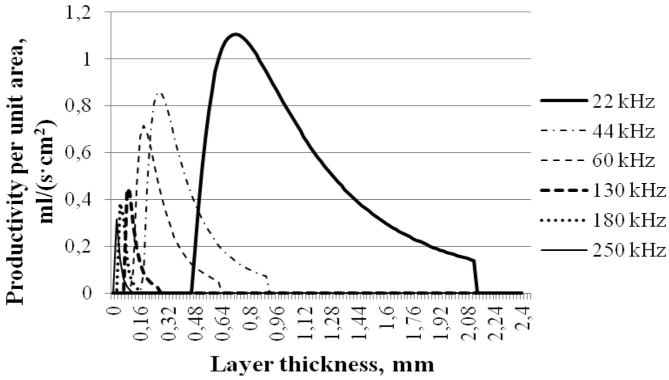
Relative productivity (the mass of atomized liquid from the surface of unit area) is defined by the formula:

$$\Pi = Vf N_S = a\frac{\lambda^2 A}{2\pi}\left(\frac{\pi^2}{2} - 2\right) f N_S, \qquad (46)$$

where N_S is a number of capillary waves on unit area of the surface, which is taken equal to the number of cavitation bubbles, *f* is a vibration frequency of working tool.

Volume content of cavitation bubbles N_V is $15 \cdot 10^6$ cm^{-3}[8] in the mode of developed cavitation that was determined experimentally in large technological volume taking inot consideration the formation of secondary nuclei of cavitation at the collapse of bubbles. As radii of cavitation bubbles are comparable with the thickness of liquid layer, the formation of secondary nuclei may not be taken into account, as volume content of nuclei is in 10^5 times less, i.e. about 150 for water in one cubic centimeter. Thus the number of cavitation bubbles in one centimeter of area N_S at the thickness of liquid layer applied for the atomization (no more than 2 mm) equals to 20 [8].

The dependences of atomization productivity on the thickness of liquid layer are obtained (see Fig. 4), optimum thickness of liquid layer (at which the productivity is maximum) coincides with the thickness of layer, at which the amplitude of capillary waves is maximum, as productivity depends linearly on the amplitude of waves.



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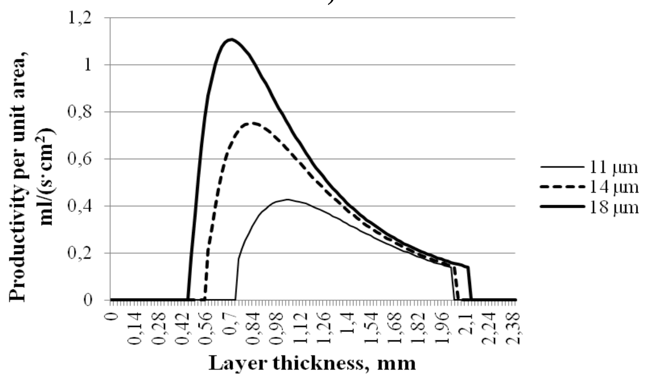

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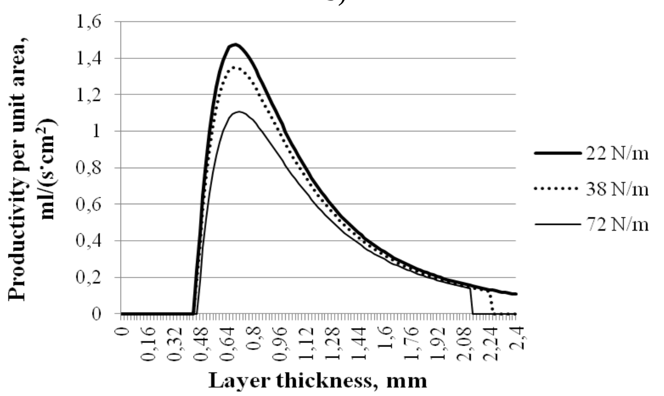

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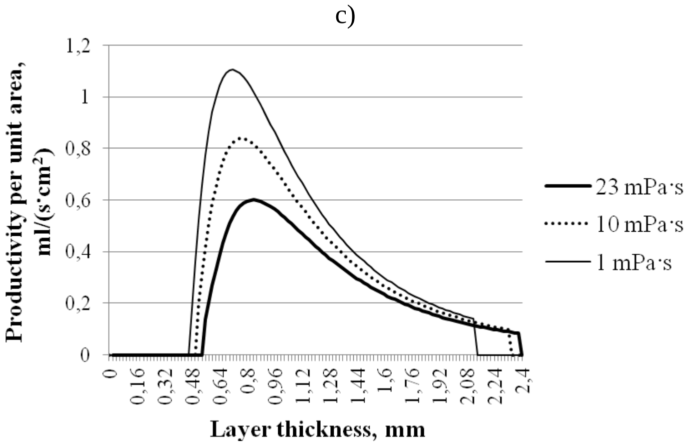

d)

Fig. 4. The dependences of atomization productivity on the thickness of liquid

layer at different parameters: frequencies (a), amplitudes (b), surface tensions (c), viscosities (d)

From the dependences presented in Fig. 4 it follows, that the most influence on the thickness of liquid layer has an amplitude, a vibration frequency and a viscosity of liquid. Surface tension does not influence on the thickness of layer. All dependences are obtained for the value of the coefficient *a=1*.

Presented in Fig. 4a dependence of atomization productivity at different frequencies of ultrasonic influence agrees with well-known fact, that the value of atomization productivity decreases with the rise of frequency. However, even at the frequency of 250 kHz the value of relative productivity of atomization achieves the values, which are enough for for practical use (~ 0.3 ml/(sec·cm^2), i.e. the increase of the frequency in 10 times from 22 kHz upto 250 kHz leads to the reduction of relative productivity of the atomization less than in 5 times from 1.1 upto 0.3 ml/sec/cm^2). It proves the necessity of development of the ultrasonic vibrating systems with the operating frequencies of upto 250 kHz for atomizing of liquids.

Described above results of theoretical study let determine optimum modes of ultrasonic influence providing the formation of drops with specified disperse characteristics.

V. Velocity of Drop Detachment And Height of Spray

For experimental determining of dependence of the thicknes of atomizing liquid layer (at which maximum productivity of atomization is provided) on the parameters of ultrasonic influence and properties of liquid there is a need in one parameter providing the most simplicity and obviousness of the measurement.

Such parameter is the height of formed spray provided at the atomization, which is directed vertically up. Its measurement is the easiest, and as it will be shown further the height of formed spray and atomization productivity achieve maximum value at same thickness of atomized liquid layer. Velocity obtained by drops at their detachment from capillary waves mainly influences on the height of the spray. As the kinetic energy of liquid in the stage of maximum development of capillary wave transforms into the energy of surface tension, the most suitable is the use of the following expression for the kinetic energy of capillary wave in the moment of drop detachment:

$$E(t_0)=\int_0^{\frac{\lambda}{2}} 2\sigma\pi x\sqrt{1+\left(\frac{2\pi}{\lambda}A\right)^2\sin^2\left(\frac{2\pi x}{\lambda}\right)}\,\partial x\cos^2(\omega t_0) \qquad (47)$$

As a result of transformation following expression for the velocity of drop detachment is obtained (48):

$$v=\sqrt{\frac{3\sigma\lambda A}{8D^3\rho}}\cos\left(\omega\sqrt{\frac{\rho\lambda^3}{64\sigma}}\right) \qquad (48)$$

For the definition of the height of the spray the second Newton law is used:

$$\frac{4}{3}\rho\pi r^3\frac{\partial u}{\partial t}=-\frac{C_X\rho_в\pi r^2u^2}{2}-\frac{4}{3}\rho\pi r^3 g \qquad (49)$$

where ρ is a density of drop liquid, u is a velocity of liquid motion, r is a radius of the drop, $\rho_в$ is a density of air, C_X is a coefficient of streamlining, which for the sphere equals to 0.3, g is a free fall acceleration.

The dependences of the height of the spray on different parameters are shown in Fig. 5.

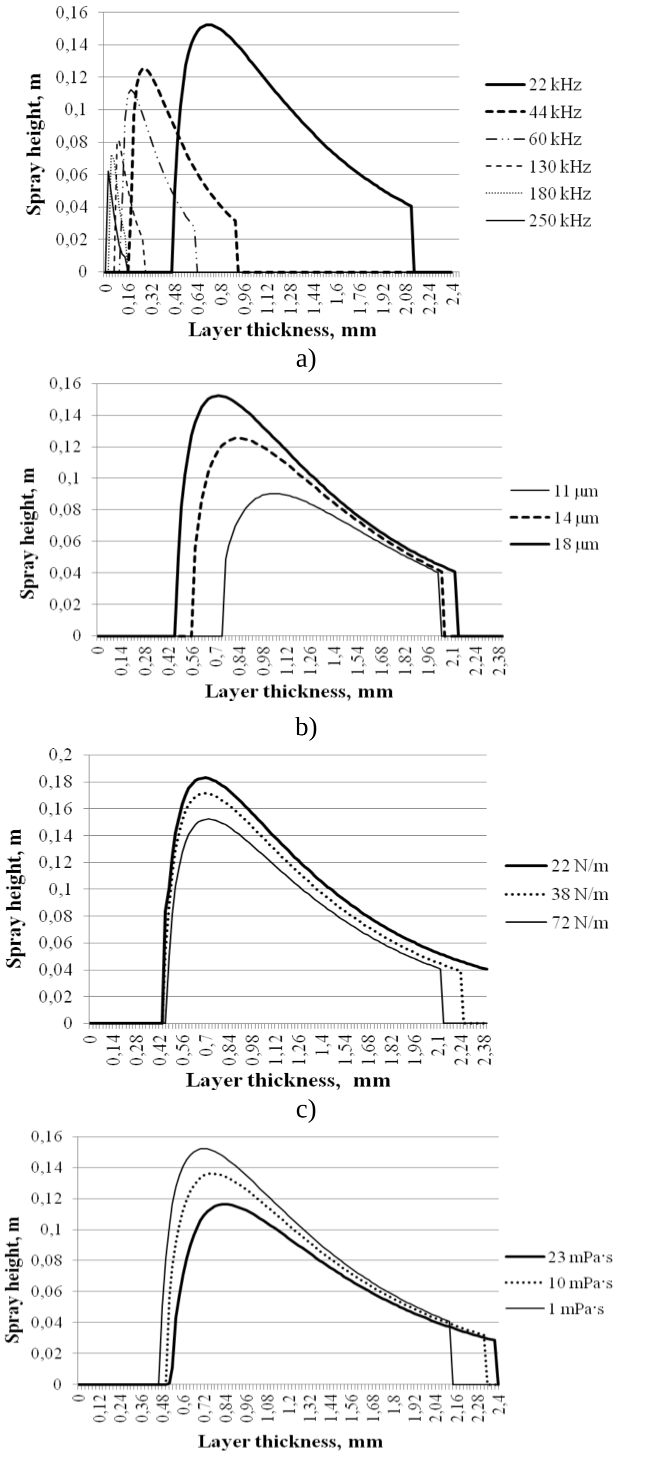

Fig. 5. The dependences of the height of the spray on the thickness of liquid layer at different parameters: frequencies (a), amplitudes (b), surface tensions (c), viscosities (d)

Optimum thickness of liquid layer agrees with the thckness, at which the height of the spray is maximum. Obtained values of the height of the spray correspond to the velocity of drop detachment of 1.1 m/sec at the frequency of 130 kHz. Presented model can be used for determining of the chracteristics of formed spray as a whole.

VI. CONCLUSION

In the paper complex mathematical formulation of the processes occuring at the formation and the propagation of ultrasonic vibrations in thin liquid layers at the interface with gaseous medium is proposed.

Developed mathematical formulation of cavitation ultrasonic atomization of liquid in a thin layer for the first time lets define the values of atomization productivity and the size of formed drops at the changes of liquid properties and parameters of ultrasonic influence.

For the first time theoretical justification of the dependences of the diameter of drops of atomized liquid on the vibration amplitude and liquid viscosity is presented. The dependences of value of atomization productivity on the thickness of this layer at different parameters of ultrasonic influence for different in properties liquids are obtained.

Revealed modes of influense allow to design specialized ultrasonic atomizers for the formation of the aerosol with specified disperse characteristics and productivity. Obtained theoretical results let formulate a number of practical recommendations on the design of ultrasonic atomizers:

1) To provide the possibility of the regulation of drop diameter it is enough to choose single operating frequency of the ultrasonic transducer and to change only its amplitude (e.g., at 22 kHz it is possible to vary the drop diameter with in the limits of 90-180 μm, at 44 kHz - 60-120 μm, at 130 kHz – 20-50 μm). It allows to avoid designing of multifrequency working tools, as the atomization is possible at the operation of the transducer at its proper resonance frequency.

2) To obtain aerosols with high variation of drop sizes (standard deviation) it is necessary to choose higher vibration frequency at high amplitudes, as it is known, that the dispersivity increases with the rise of frequency. The theory shows, that it is possible to obtain the same mean diameter of drops at different frequencies at rather high amplitudes.

3) At the development of the system of automatic determination and maintenance of the thickness of layer of atomized liquid it is acceptable to ignore surface tension of liquid, but take into account its viscosity, that essentially simlifies the task of the design.

4) For the atomization of more viscous liquids it is necessary to choose lowered frequency or higher vibration amplitude to obtain specified mean diameter of drops.

5) For experimental determination of optimum thickness of the layer, at which the productivity is maximum, it is possible to measure only the height of the spray, that is necessary for the development of the system of automatic maintenance of optimum thickness.

6) For fine-dispersed atomization it is practical to design high-frequency vibrating systems (up to 250 kHz), as the productivity does not change in the range of frequencies of 130-250 kHz, at that the drop diameter essentially reduces.

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